

# Design and Simulation of New Inverse Kinematic Algorithm to Manipulate a 5-DOF Humanoid Robotic Arm

Ammar A. Al-Hamadani <sup>1</sup>, Maad Kamal Al-Anni <sup>2</sup>, Ayad Mahmood Kwad <sup>3</sup>, Hugot Pichon <sup>4</sup>, Gamil R. S. Qaid <sup>5</sup>,  
Najran Nasser Hamood <sup>6</sup>

<sup>1</sup> Department of Computer Engineering, College of Engineering, Al-Iraqia University, Baghdad- Iraq  
Email: ammar.aladin@aliraqia.edu.iq

<sup>2</sup> Aix Marseille University, CNRS, ENSAM, Universit'e De Toulon, LIS UMR 7020, 13397 Marseille, France  
Email: alannimaad@gmail.com

<sup>3</sup> Department of Electrical Engineering, College of Engineering, Al-Iraqia University, Baghdad- Iraq  
Email: ayad.m.k@aliraqia.edu.iq

<sup>4</sup> Laboratoire d'Astrophysique de Marseille (LAM), Aix-Marseille Universit'e, CNRS, CNES, Marseille, France  
Email: hugot.pichon@g2elab.grenoble-inp.fr

<sup>5</sup> Faculty of Computer Science and Engineering, Hodeidah University, Al Hudaydah -Yemen  
Email: dr.g\_qaid@hoduniv.net.ye

<sup>6</sup> Faculty of Computer Science and IT, Sana'a University, Sana'a-Yemen  
Email: n.aldawla@su.edu.ye

## Article History

Received: Mar. 04, 2026

Revised: May. 09, 2026

Accepted: Jun. 08, 2026

## Abstract

In this paper, a new solution of inverse kinematics was derived and programmed as an algorithm. The algorithm was coded and embedded in a MATLAB simulation program to manipulate a 5-DOF Humanoid Robotic Arm (HRA) and assess its reaching accuracy. The algorithm was designed as follows: joint frames were modelled based on the Denavit-Hartenberg (D-H) concept. Configuration for each joint frame was designed using the proposed D-H parameters. Forward kinematic (FK) equations were derived using the designed frames configuration. The inverse kinematic problem was solved (derived) using an analytical method to find five joint angles for the desired location and orientation. IK equations were derived from the FK transformation matrix for the given location and orientation to reverse back the joint angles. The Graphical User Interface (GUI) was designed using MATLAB to simulate the proposed FK/IK algorithm. A group of desired locations was handled using the GUI to show the resultant pose of the HRA. The accuracy of the proposed IK was assessed by means of positional error. The lowest and highest average positional errors achieved were (0.026 and 0.79) cm, respectively.

**Keywords-** Inverse Kinematics; Humanoid Robotic Arm; Analytical Solution; Graphical User Interface.

## I. INTRODUCTION

Kinematics deals with speed or velocity and does not cover forces which affect the movement. Kinematics can be direct (or forward) and inverse. Forward Kinematics (FK) — calculating the position and orientation of the last segment in a kinematics chain by defining angles for every joint. Inverse Kinematics: In contrast, inverse kinematics calculates the angles of a chain of joints according to the position and orientation of the last segment in this chain [1]. The forward kinematics problem is considered as direct forward and there is often a solution for a particular robotic manipulator. In contrast, the inverse kinematic solution is regarded as computationally expensive and has singularities and nonlinear problems that make the solution more difficult. Hence, a complete analytical solution exists for only a few kinds of simple manipulators [2]. In general, there are two solutions for the inverse kinematic problem, numerical and analytical solutions. In the first method, the joint angle values are obtained according to numerical techniques, and in the second method, geometric and algebraic solutions are utilized [3].

In order to model a joint, it must first be possible to represent the specific joint frame of each bone in a given pair such that its state can be described in parametric form. We need to define the reference and relative coordinates on the reference bone and the moving bone, respectively. The DoF for the joint can then be represented through a composition of translation in Cartesian space and rotation about all the angular coordinate axes. Frame rotation can be defined in many concepts of them, Denavit-Hartenberg (D-H) technique. This technique models the robotic arm as joints interconnected by links to represent the joint motion transformation. In this technique, a reference frame is assigned to each joint then this frame is transformed from the first joint to the next until reaching the last. Then, all transformations are combined to have the arm's total transformation matrix [4].

In the field of robotic arm manipulators, many kinematics models were adopted in the literature to perform the arm movements. Researchers in [5] proposed an algorithm to solve the IK problem for a complex wrist structure with a 6-DOF robotic manipulator. The algorithm was based on the concept of a rotation vector and it has been tested practically on the M2000 robotic manipulator. In [6], it has been proposed an algorithm for the IK computation model designed for 7-DOF virtual human arm suitable for reaching tasks performed by interactive and autonomous virtual human. The algorithm consisted of two phases, the first is to set the limitations of the real task, the second is the analytical solution of IK using trigonometric relations and algebraic solutions based on the joints' limitations. It has been proposed in [7] a method to solve the IK problem for a 7-DOF human arm that combines the analytical solution of IK with a nonlinear optimization algorithm. The analytical solution has been used to reduce the problem size from seven joint variables to a single variable and the nonlinear optimization algorithm has been used to find the approximate solution thus reducing both computation time and error. Researchers in [8] proposed an algorithm to solve IK problem for TQ MA2000 6-DOF robotic manipulator with wrist offset. The authors proposed to use a strategy in which the algorithm starts to find the wrist point using vector computation followed by computing the first three joint angles and then computing the wrist angle using an analytical solution. In [9], a mathematical model has been proposed to solve the FK and IK problem for a 5-DOF robotic manipulator. The algorithm has been derived based on the geometric method and Denavit-Hartenberg (D-H) parameter configuration scheme.

In this article, it's proposed to find IK solution for a robotic manipulator defined as a 5-DOF Humanoid Robotic Arm (HRA) represented by five joints and two links. The proposed method to model the motion of the HRA is defined by the Denavit-Hartenberg (D-H) technique. D-H method used to derive the kinematic equations for the proposed HRA, where the arm is treated as a serial joints manipulator with its pose defined by a matrix representing the position and orientation of one joint with respect to the previous one. For the 5-DOF arm, it's required to solve the Inverse Kinematic (IK) problem in order to find the five joint angles ( $\theta_1, \theta_2, \dots, \theta_5$ ) and achieve the desired position and orientation of the arm's End-Effector (EE). To the IK problem, an analytical solution is proposed in which joint angles are represented with equations derived from the FK matrix using algebraic, geometrical, and trigonometric relations. It was aimed to use the proposed algorithm with the BCI paradigm discussed in [10] and extend it to manipulate a robotic arm in real time.

## II. PROPOSED METHOD

### A. Proposed Forward Kinematics

By considering the HRA shown in Fig. 1 that modeled as 5-DOF, it can be seen that the shoulder is represented by three intersecting revolute joints, the elbow is represented by one revolute joint, and the wrist is represented by one revolute joint.

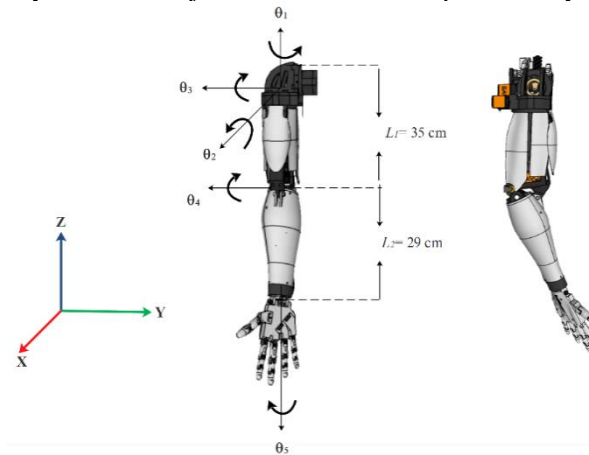


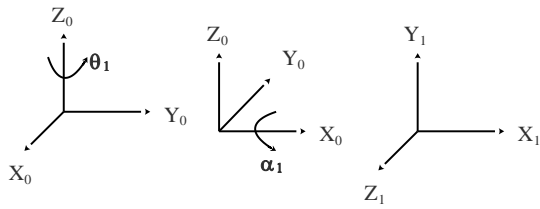
Figure 1. Proposed DOFs for the HRA

Based on the D-H concept, the proposed model of D-H parameter was constructed as in Fig. 2 below.

#### Frame (0-1):

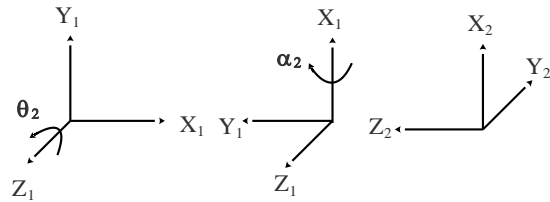
Starting from the shoulder joint,

$$\theta_1 = \theta_1 + \frac{\pi}{2}, d_1 = 0, a_1 = 0, \alpha_1 = \frac{\pi}{2}$$



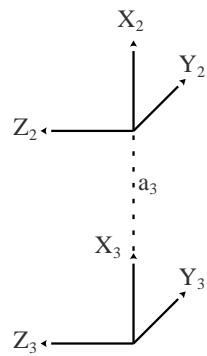
**Frame (1-2):**

$$\theta_2 = \theta_2 + \frac{\pi}{2}, d_2 = 0, a_2 = 0, \alpha_2 = -\frac{\pi}{2}$$



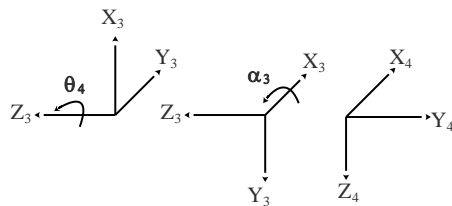
**Frame (2-3):**

$$\theta_3, d_3 = 0, a_3 = -35 \text{ cm}, \alpha_3 = 0$$



**Frame (3-4):**

$$\theta_4 = \theta_4 + \frac{\pi}{2}, d_4 = 0, a_4 = 0, \alpha_4 = -\frac{\pi}{2}$$



**Frame (4-5):**

$$\theta_5, d_5 = 29 \text{ cm}, a_5 = 0, \alpha_5 = 0$$

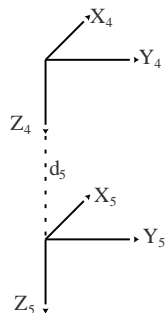


Figure 2. Modeling of D-H parameters to construct HRA's frames of motion.

From Fig. 2, the designed D-H parameters are concluded in Table 1, which is called the D-H table and hence,  $a_3 = -L_1 = -35cm$  and  $d_5 = L_2 = 29cm$ .

TABLE 1. PROPOSED D-H PARAMETERS TABLE.

Frame No.	$\theta$	$d$	$a$	$\alpha$
0-1	$\theta_1 + \frac{\pi}{2}$	0	0	$\frac{\pi}{2}$
1-2	$\theta_2 + \frac{\pi}{2}$	0	0	$-\frac{\pi}{2}$
2-3	$\theta_3$	0	$a_3$	0
3-4	$\theta_4 + \frac{\pi}{2}$	0	0	$-\frac{\pi}{2}$
4-5	$\theta_5$	$d_5$	0	0

These parameters were utilized to describe the position and orientation of the joint-link using the following transformation matrices:

$$A_i = \begin{bmatrix} C \theta_i & -S \theta_i C \alpha_i & S \theta_i S \alpha_i & a_i C \theta_i \\ S \theta_i & C \theta_i C \alpha_i & -C \theta_i S \alpha_i & a_i S \theta_i \\ 0 & S \alpha_i & C \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Keeping in mind that  $\cos\left(\theta + \frac{\pi}{2}\right) = -\sin(\theta)$  and  $\sin\left(\theta + \frac{\pi}{2}\right) = \cos(\theta)$ , the FK matrices of joint frames are:

$$A_1 = \begin{bmatrix} -S_1 & 0 & C_1 & 0 \\ C_1 & 0 & S_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$A_2 = \begin{bmatrix} -S_2 & 0 & -C_2 & 0 \\ C_2 & 0 & -S_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3 C_3 \\ S_3 & C_3 & 0 & a_3 S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$A_4 = \begin{bmatrix} -S_4 & 0 & -C_4 & 0 \\ C_4 & 0 & -S_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$A_5 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

Where  $S$  is referred to  $\sin(\theta)$  and  $C$  is referred to  $\cos(\theta)$ . The total FK transformation matrix from joint one to joint five ( $T_5^1$ ) can be calculated as in Eq.(7):

$$T_5^1 = A_1 A_2 A_3 A_4 A_5 \quad (7)$$

The transformation  $T_5^1$  represents the actual orientation and position of the EE. Therefore, Eq.7 can be considered as follows:

$$T^a = T_5^1 = A_1 A_2 A_3 A_4 A_5 = \begin{bmatrix} n_x^a & o_x^a & a_x^a & P_x^a \\ n_y^a & o_y^a & a_y^a & P_y^a \\ n_z^a & o_z^a & a_z^a & P_z^a \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

Where  $T^a$  is the actual transformation of the EE,  $n^a, o^a, a^a$  are the rotation parameters and  $P^a$  is the position parameters of the last frame, which represent the EE of the HRA. The symbolic expression for each element of the resultant matrix in Eq. (8) can be viewed in detail as follows:

$$n_x^a = C_5(S_4(C_1S_3 - C_3S_1S_2) - C_4(C_1C_3 + S_1S_2S_3)) - C_2S_1S_5 \quad (9)$$

$$n_y^a = C_5(S_4(S_1S_3 + C_1C_3S_2) - C_4(C_3S_1 - C_1S_2S_3)) + C_1C_2S_5 \quad (10)$$

$$n_z^a = S_2S_5 - C_5C_2(C_3S_4 + C_4S_3) \quad (11)$$

$$o_x^a = -S_5(S_4(C_1S_3 - C_3S_1S_2) - C_4(C_1C_3 + S_1S_2S_3)) - C_2C_5S_1 \quad (12)$$

$$o_y^a = C_1C_2C_5 - S_5(S_4(S_1S_3 + C_1C_3S_2) - C_4(C_3S_1 - C_1S_2S_3)) \quad (13)$$

$$o_z^a = C_5S_2 + S_5C_2(C_3S_4 + C_4S_3) \quad (14)$$

$$a_x^a = S_4(C_1C_3 + S_1S_2S_3) + C_4(C_1S_3 - C_3S_1S_2) \quad (15)$$

$$a_y^a = S_4(C_3S_1 - C_1S_2S_3) + C_4(S_1S_3 + C_1C_3S_2) \quad (16)$$

$$a_z^a = C_2S_3S_4 - C_2C_3C_4 \quad (17)$$

$$P_x^a = d_5(S_4(C_1C_3 + S_1S_2S_3) + C_4(C_1S_3 - C_3S_1S_2)) - a_3C_1S_3 + a_3C_3S_1S_2 \quad (18)$$

$$P_y^a = d_5(S_4(C_3S_1 - C_1S_2S_3) + C_4(S_1S_3 + C_1C_3S_2)) - a_3S_1S_3 - a_3C_1C_3S_2 \quad (19)$$

$$P_z^a = C_2(a_3C_3 - d_5(C_3C_4 - S_3S_4)) \quad (20)$$

### B. Proposed Analytical Solution for the Inverse Kinematic (IK) Problem

Equations 9 - 20 represent the FK parameters of the humanoid robotic arm in terms of its five joint angles. Such parameters reflect the actual position and orientation of the EE. The solution arrangement is based on equating the desired transformation matrix  $T^d$  given by Eq. 21 (provided by the user) with the actual transformation matrix given by Eq. 8. Then, angle decoupling is performed by directly inverting the FK equation through pre-multiplying both sides by  $A_1^{-1}$ . The calculation of joint angles is going as follows:

At first, to simplify the solutions, it was assumed that the value of  $\theta_5$  is always equal to zero since it represents the end-effector (EE) of the HRA and it's assumed that its value doesn't affect the pose of the HRA. Suppose that the desired transformation matrix is:

$$T^d = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (21)$$

Now, to perform the inverse kinematics, let's equate Eq. 21 with Eq. 8 to yield:

$$\begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_1 A_2 A_3 A_4 A_5 \quad (22)$$

### Calculation of $\theta_1$ :

To calculate  $\theta_1$ , FK inverting is performed by pre-multiplying both sides of Eq. 22 with  $A_1^{-1}$ , that's will eliminates  $A_1$  from the right side of Eq.22 as follows:

$$A_1^{-1} \times \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_1^{-1} \times A_1 A_2 A_3 A_4 A_5 = A_2 A_3 A_4 A_5 \quad (23)$$

Therefore,

$$\begin{bmatrix} C_1 n_y - n_x S_1 & C_1 o_y - o_x S_1 & a_y C_1 - a_x S_1 & P_y C_1 - P_x S_1 \\ n_z & o_z & a_z & P_z \\ C_1 n_x + n_y S_1 & C_1 o_x + o_y S_1 & a_x C_1 + a_y S_1 & P_x C_1 + P_y S_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_2 S_5 + C_5 (C_3 S_2 S_4 + C_4 S_2 S_3) & C_2 C_5 - S_5 (C_3 S_2 S_4 + C_4 S_2 S_3) & C_3 C_4 S_2 - S_2 S_3 S_4 & d_5 (C_3 C_4 S_2 - S_2 S_3 S_4) - a_3 C_3 S_2 \\ S_2 S_5 - C_5 (C_2 C_3 S_4 + C_2 C_4 S_3) & C_5 S_2 + S_5 (C_2 C_3 S_4 + C_2 S_4 S_3) & C_2 S_3 S_4 - C_2 C_3 C_4 & a_3 C_2 C_3 - d_5 (C_5 C_3 C_4 - C_2 S_3 S_4) \\ -C_5 (C_3 C_4 - S_3 S_4) & S_5 (C_3 C_4 - S_3 S_4) & C_3 S_4 + C_4 S_3 & d_5 (C_3 S_4 + C_4 S_3) - a_3 S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (24)$$

Equating the matrix elements (3, 2) of both sides of Eq. 24, it provides:

$$C_1 o_x + o_y S_1 = S_5 (C_3 * C_4 - S_3 S_4) \quad (25)$$

Since  $\theta_5$  assumed to be zero, then  $\sin(0)$  equals zero, which means that  $S_5$  is zero. Therefore, Eq. 25 would be:

$$C_1 o_x + o_y S_1 = 0 \rightarrow \theta_1 = \tan^{-1}\left(\frac{-o_x}{o_y}\right) \quad (26)$$

### Calculation of $\theta_2$ :

The next step is to find  $\theta_2$  by pre-multiplying both sides of Eq. 22 this time with  $A_2^{-1}A_1^{-1}$ :

$$A_2^{-1}A_1^{-1} \times \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_3 A_4 A_5 \quad (27)$$

Which yields:

$$\begin{bmatrix} C_2 n_z - C_1 n_y S_2 + n_x S_1 S_2 & C_2 o_z - C_1 o_y S_2 + o_x S_1 S_2 & a_z C_2 - a_y C_1 S_2 + a_x S_1 S_2 & P_z C_2 - P_y C_1 S_2 + P_x S_1 S_2 \\ -C_1 n_x - n_y S_1 & -C_1 o_x - o_y S_1 & -a_x C_1 - a_y S_1 & -P_x C_1 - P_y S_1 \\ C_2 n_x S_1 - C_1 n_y - n_z C_2 S_2 & C_2 o_x S_1 - C_1 o_y - o_z S_2 & a_x C_2 S_1 - a_y C_1 C_2 - a_z S_2 & P_x C_2 S_1 - P_y C_1 C_2 - P_z S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -C_5 (C_3 S_4 + C_4 S_3) & S_5 (C_3 S_4 + C_4 S_3) & S_3 S_4 - C_3 C_4 & a_3 C_3 - d_5 (C_3 C_4 - S_3 S_4) \\ C_5 (C_3 C_4 - S_3 S_4) & -S_5 (C_3 C_4 - S_3 S_4) & -C_3 S_4 - C_4 S_3 & a_3 S_3 - d_5 (C_3 S_4 + C_4 S_3) \\ -S_5 & -C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (28)$$

Equating (3, 4) matrices elements of both sides of Eq. 28 gives us:

$$P_x C_2 S_1 - P_y C_1 C_2 - P_z S_2 = 0 \rightarrow C_2 (P_x S_1 - P_y C_1) = P_z S_2 \rightarrow (P_x S_1 - P_y C_1) = P_z \left(\frac{S_2}{C_2}\right) \rightarrow \theta_2 = \tan^{-1}\left(\frac{P_x S_1 - P_y C_1}{P_z}\right) \quad (29)$$

Noting that,  $S_1$  and  $C_1$  can be found using  $\theta_1$  calculated by Eq.26.

### Calculation of $\theta_3$ :

It was found that the best way to solve for  $\theta_3$  is by starting the derivation from Eq. 22 as follows:

$$\begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_1 A_2 A_3 A_4 A_5 \quad (30)$$

Knowing that  $[A_1 A_2 A_3 A_4 A_5]$  equals the transformation matrix defined by Eqs. (9)-(20). Therefore, equating (3, 3) and (3, 4) matrices elements on both sides and performing some algebraic operations, are performed as follows: Equating (3, 3) elements yields,

$$a_z = C_2 S_3 S_4 - C_2 C_3 C_4 \rightarrow a_z = C_2 (S_3 S_4 - C_3 C_4) \quad (31)$$

Based on the trigonometric identity ( $\cos x \cos y - \sin x \sin y = \cos(x + y)$ ) then,  $(S_3 S_4 - C_3 C_4) = -C_{34}$  so, the substitution of this quantity in Eq. 4.35 yields:

$$a_z = -C_2 C_{34} \tag{32}$$

Equating (3, 4) elements yields,

$$P_z = a_3 C_2 C_3 - d_5 (C_2 C_3 C_4 - C_2 S_3 S_4) \rightarrow P_z - a_3 C_2 C_3 = -d_5 C_2 (C_3 C_4 - S_3 S_4) \tag{33}$$

Using the same trigonometric identity above, where  $(C_3 C_4 - S_3 S_4) = C_{34}$ ,

$$P_z - a_3 C_2 C_3 = -d_5 C_2 C_{34} \tag{34}$$

Dividing Eq. 32 by Eq. 34 results in:

$$\frac{a_z}{P_z - a_3 C_2 C_3} = \frac{-C_2 C_{34}}{-d_5 C_2 C_{34}} \rightarrow \frac{a_z}{P_z - a_3 C_2 C_3} = \frac{1}{d_5} \tag{35}$$

$$P_z - a_3 C_2 C_3 = a_z d_5 \rightarrow C_3 = \frac{P_z - a_z d_5}{a_3 C_2} \tag{36}$$

Where,  $P_z$ ,  $a_z$  are parameters taken from the desired transformation matrix and  $a_3$ ,  $d_5$  are taken from the D-H parameters table. Knowing that,  $\sin x = \sqrt{1 - \cos^2 x}$  then,

$$S_3 = \sqrt{1 - C_3^2} \tag{37}$$

Then,

$$\theta_3 = \tan^{-1} \left( \frac{S_3}{C_3} \right) \tag{38}$$

**Calculation of  $\theta_4$ :**

The angle of  $\theta_4$  represents the elbow position as shown in Fig. 3. It can be calculated geometrically; the line represents the distance between the shoulder and the end-effector (EE) was termed as (SE). This distance is defined as below:

$$SE = \sqrt{(P_x^2 + P_y^2 + P_z^2)} \tag{39}$$

Where,  $(P_x, P_y, P_z)$  represents the desired position given in the desired transformation matrix. According to Fig. 4, SE can be found in another way, such as:

$$SE^2 = a_3^2 + d_5^2 + 2 a_3 d_5 \cos(\pi - \theta_4) \tag{40}$$

$$\theta_4 = \pi - \cos^{-1} \left( \frac{a_3^2 + d_5^2 - SE^2}{2 a_3 d_5} \right) \tag{41}$$

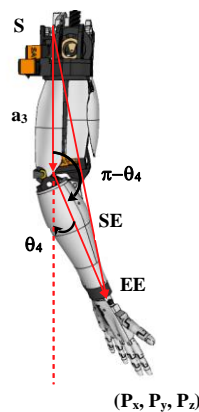


Figure 3. Proposed modeling for the elbow position of the HRA.

Based on the assumption made at the beginning of this section, it was regarded that the value of  $\theta_5$  is zero since (joint 5) represents the wrist or the EE of the HRA. Its rotation represents the rotation of the robotic hand about the joint's Z-axis and no further link exists after that. In other words, no transformation of motion existed after joint 5, either. After the calculation of all angles  $(\theta_1, \dots, \theta_5)$ , the accuracy of the proposed IK algorithm can be checked by calculating the FK using these angles, then calculating the absolute error between the resultant actual position and the desired position as follows:

Using calculated angles, find the new actual transformation matrix:

$$T_5^1 = A_1 A_2 A_3 A_4 A_5 = \begin{bmatrix} n_x^a & o_x^a & a_x^a & P_x^a \\ n_y^a & o_y^a & a_y^a & P_y^a \\ n_z^a & o_z^a & a_z^a & P_z^a \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (42)$$

$$\begin{bmatrix} err_x \\ err_y \\ err_z \end{bmatrix} = \begin{bmatrix} P_x - P_x^a \\ P_y - P_y^a \\ P_z - P_z^a \end{bmatrix} \quad (43)$$

Where,  $[err_x, err_y, err_z]^T$  is the absolute error vector in x, y, z axes respectively,  $[P_x, P_y, P_z]^T$  is the desired position vector, and  $[P_x^a, P_y^a, P_z^a]$  is the actual position vector. Finally, the flowchart of the proposed inverse kinematic algorithms used to control a 5-DOF humanoid robotic arm is represented in Fig. 4. Note that this flowchart is used in the programming of the IK algorithm in MATLAB. A Graphical User Interface (GUI) was developed based on MATLAB to compute the forward and inverse kinematics for the proposed HRA. Based on the previous analysis, the user can compute the FK and IK solution. The algorithm represented in the flowchart of Fig. 4 was used to generate the joint's motion (joint angles). A motion simulator where the HRA is represented as a 3D arm with 5-DOF was also included on the right side of the GUI to show the generated joint angles in the form of motion. The GUI simulator will simulate the HRA's movement for a given set of joint angles or location values. When the value of the angles are manually entered into the corresponding fields of (Q1 to Q5) and the Forward button is clicked, a transformation matrix  $T_5^1$  representing the new end effector's position and orientation will be generated. When the desired location values were entered into ( $P_x, P_y, P_z$ ) fields and the Inverse button was clicked, a set of five angle values was generated and shown in the (Q1 to Q5) fields.

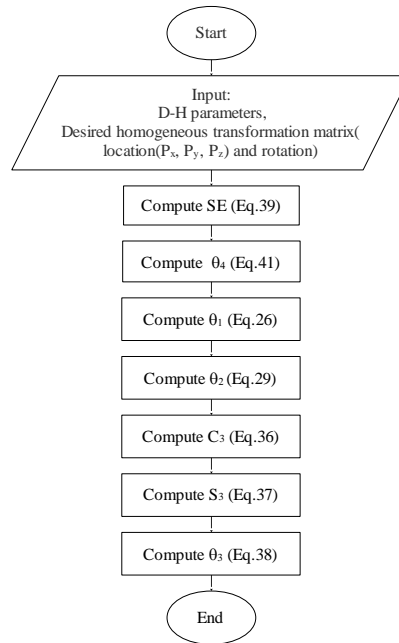


Figure 4. Flowchart of the proposed IK algorithm using analytical solution.

### III. RESULT AND DISCUSSION

In this article, a new mathematical model with kinematic analysis of a 5-DOF humanoid robotic arm was presented. The proposed HRA was modeled based on the D-H technique and the FK / IK solutions were implemented and generated by the developed MATLAB code. For all calculations, the following joint constraints shown in Table 2 were followed and any joint angle outside these constraints is excluded. Hence, all the angles used with the proposed IK algorithm are in radian form.

TABLE 2. JOINT ANGLES CONSTRAINTS.

Joint	Range of usable angles
1	$-90 \leq Q1 \leq 90$
2	$-90 \leq Q2 \leq 0$
3	$0 \leq Q3 \leq 180$
4	$0 \leq Q4 \leq 90$
5	$Q5 = \text{all zero}$

*A. Results of Forward kinematics*

A typical case was implemented and calculated using the generated MATLAB code, along with the GUI was included here to show the achieved results. The initial position of the HRA is given in the matrix presented in Eq. 44, where all the values of joint angles (Q1 to Q5) were zeros as shown in Fig. 5.

$$Initial\ Position = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -64 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{44}$$

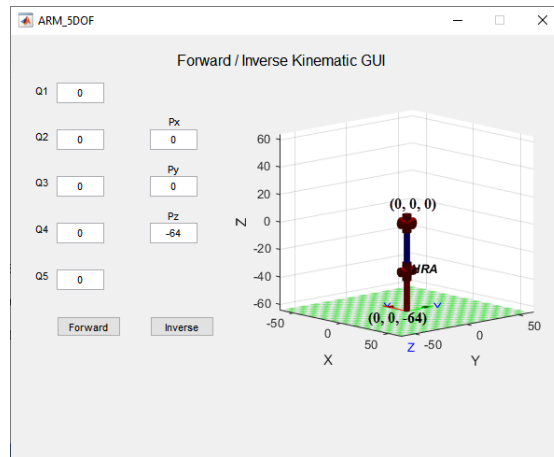


Fig. 5. Initial position of the HRA.

Changing the values of thetas from their initial position to the new values given in Table 3, the Forward kinematics transformation matrices are generated using Eqs. (2-6). The total transformation from the base frame to the End-Effector is represented by the total transformation matrix  $T_5^1$  calculated using Eq. (7).

TABLE 3. SET OF JOINT ANGLES WITH D-H PARAMETERS.

joint	$\theta$ (degree)	d (cm)	a (cm)	$\alpha$ (degree)
1	45	0	0	90
2	0	0	0	-90
3	30	0	-35	0
4	50	0	0	-90
5	0	29	0	0

$T_5^1$  was determined also using the developed GUI by entering the desired joint angles into (Q1 to Q5) fields, and clicking the forward button. The same result of the EE position as in column three of Eq. (45) was achieved and displayed in the ( $P_x, P_y, P_z$ ) fields of the GUI window, see Fig. 6.

$$T_5^1 = \begin{bmatrix} -0.1228 & -0.7071 & 0.6964 & 32.5689 \\ -0.1228 & 0.7071 & 0.6964 & 32.5689 \\ -0.9848 & 0 & 0.1736 & -35.3467 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{45}$$

*B. Results of Inverse Kinematics*

To obtain the required joint angles from a given position and orientation. The derived equations were used to calculate the necessary joint angles of a given position and orientation, as depicted in the flow chart shown in Fig. 4. For each case, a number of target positions with the respective orientations are combined to search for suitable joint angles. Then, the IK algorithm error was computed by using Eq. (43). In this step, we fed the joint Angles yielded from the IK solution into an FK algorithm to find its actual position. At last, this calculated position was compared with the intended position to assess how accurate the IK algorithm is based on the error in the resulting position. Only two cases representing good and higher error results are displayed to show the algorithm's limits, as shown in Tables 4 and 5, respectively.

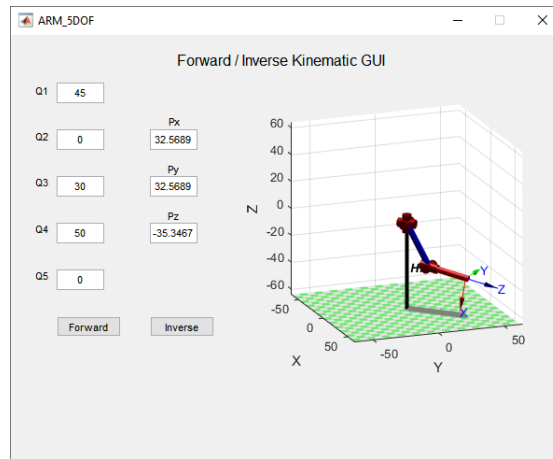


Fig. 6. Set of joint angles applied in FK algorithm with its corresponding HRA movement.

TABLE 4. FIRST CASE REPRESENTING GOOD RESULT OF IMPLEMENTING THE PROPOSED IK ALGORITHM.

<b>Desired Position (cm)</b>	$\begin{bmatrix} 15 \\ -10 \\ -59 \end{bmatrix}$
<b>Desired Orientation</b>	$\begin{bmatrix} -0.8572 & 0 & 0.5150 \\ -0.0894 & 0.9848 & -0.1488 \\ -0.5072 & -0.1736 & -0.8441 \end{bmatrix}$
<b>Calculated Angles (degree)</b>	0.00 -9.62 0.00 31 0.00
<b>Actual Position (from FK) (cm)</b>	$\begin{bmatrix} 14.93 \\ -10 \\ -59.01 \end{bmatrix}$
<b>Position Error (cm)</b>	$\begin{bmatrix} 0.07 \\ 0 \\ 0.01 \end{bmatrix}$

TABLE 5. SECOND CASE REPRESENTING HIGHER ERROR RESULT OF IMPLEMENTING THE PROPOSED IK ALGORITHM.

<b>Desired Position (cm)</b>	$\begin{bmatrix} 21 \\ -35 \\ -48 \end{bmatrix}$
<b>Desired Orientation</b>	$\begin{bmatrix} -0.866 & 0 & 0.5 \\ -0.2939 & 0.809 & -0.509 \\ -0.4045 & -0.5878 & -0.7006 \end{bmatrix}$
<b>Calculated Angles (degree)</b>	0.00 -36.09 11.8 20.29 0.00
<b>Actual Position (from FK) (cm)</b>	$\begin{bmatrix} 22.57 \\ -34.65 \\ -47.53 \end{bmatrix}$
<b>Position Error (cm)</b>	$\begin{bmatrix} 1.57 \\ 0.35 \\ 0.47 \end{bmatrix}$

As seen in Table 4, the desired position achieves a lower position error as compared with Table 5. It's found that such a rise of error is attributable to the non-linearity of trigonometric functions ( $\sin$  and  $\cos$ ), which are used extensively in the calculation of both joint angles and rotation matrices. Entering each of the above positions into the corresponding fields of the GUI results in the calculation of the required joint angles. Simulation of the motion that HRA is supposed to do when using each of those positions is depicted in Fig. 7.

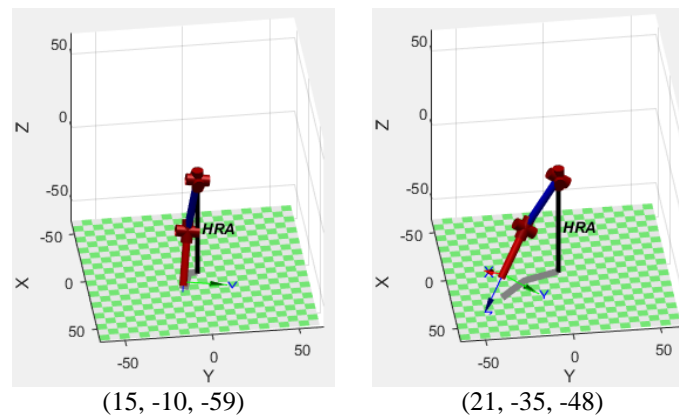


Fig. 7. GUI simulation of the HRA motion.

Average positional error can be calculated for each case by summing ( $P_x$ ,  $P_y$ ,  $P_z$ ) errors and dividing by three. An average of 0.026 cm position error was regarded as a good result as compared with a higher error of the value of 0.79 cm.

#### IV. CONCLUSION

A new analytical solution of IK was derived for five degree of freedom humanoid robotic arm. The proposed solution was programmed and simulated by means of MATLAB GUI. The IK accuracy was determined by means of absolute position error between desired and actual positions. The lowest achieved average position error was 0.026 cm for the point (15, -10, -59) while the highest average positional error was 0.79 cm for the point (21, -35, -48). The proposed algorithm can be used in the application of the Brain-Computer-Interface (BCI) to help amputated persons.

#### ACKNOWLEDGMENT

The authors would like to thank Al-Iraqi University, especially the College of Engineering, for the unlimited support, trust, and encouragement. Also, our thanks are extended to Aix-Marseille University, the Laboratoire d'Astrophysique de Marseille (LAM), Hodeidah University (Yemen), and Sana'a University (Yemen) for their valuable scientific collaboration

#### REFERENCES

- [1] H. Ye, D. Wang, J. Wu, Y. Yue, and Y. Zhou, Forward and inverse kinematics of a 5-DOF hybrid robot for composite material machining, *Robot. Comput. Integr. Manuf.* **65**(2020) (2020) 1-11
- [2] A. A. Al-hamadani, M. Z. Al-faiz, and S. Member, Inverse Kinematic Based Brain Computer Interface to Control Humanoid Robotic Arm, *Int. J. Mech. Mechatronics Eng. IJMME-IJENS* **20**(01) (2020) 15-24
- [3] J. Craig (ed.), *Introduction to robotics, mechanics and control*, 4th edn. (Pearson, London, 2017).
- [4] S. B. Niku, Kinematics of Robots: Position Analysis, in *Introduction to robotics: analysis, control, applications*, 2nd edn. (John Wiley & Sons, 2010), pp. 33-113.
- [5] M. Z. Al-Faiz, M. Z. Othman, and B. B. Al-Bahri, An algorithm to solve the inverse kinematics problem of a robotic manipulator based on rotation vectors, in *2006 IEEE GCC Conference (GCC)* (2006), pp. 1-6.
- [6] M. Z. Al-Faiz and Y. I. Al-Mashhadany, Analytical solution for anthropomorphic limbs model, (IK of human arm), in *2009 IEEE Symposium on Industrial Electronics & Applications* (2009), pp. 684-689.
- [7] M. Z. Al-Faiz, A. A. Ali, and A. H. Miry, Human Arm Inverse Kinematic Solution Based Geometric Relations and Optimization Algorithm, *International J. Comput. Sci. Secur.* **1**(3) (2011) 1-10.
- [8] M. Z. Al-Faiz and M. S. Saleh, Inverse Kinematics Analysis for Manipulator Robot with Wrist Offset Based On the Closed-Form Algorithm, *Int. J. Robot. Autom. (IJRA)* **2**(4) (2011) 256-264.
- [9] V. N. Iliukhin, K. B. Mitkovskii, D. A. Bizyanova, and A. A. Akopyan, The Modeling of Inverse Kinematics for 5 DOF Manipulator, *Procedia Eng.*, **176**(2017) (2017) 498-505.
- [10] M. Z. Al Faiz and A. A. Al-Hamadani, Online Brain Computer Interface Based Five Classes EEG To Control Humanoid Robotic Hand, in *2019 42nd International Conference on Telecommunications and Signal Processing (TSP)* (Budapest, Hungary, 2019), pp. 406-410.