

The Effectiveness of the Mutual Coupling Model, Coupled Voltage to Uncoupled Currents, in Receiving Antenna Arrays

Rabah Abduljabbar Jasem

PhD in Electrical Engineering
Email: rabah_alobaidi5@yahoo.com
<https://orcid.org/0000-0002-7533-1715>

Abstract

This work examines the efficiency of the model known as “Coupled Voltages to Uncoupled Currents” for representing the mutual coupling phenomenon between the elements of a receiving antenna array. This model postulates the relationship between the coupled-free currents induced in the elements of the array when receiving electromagnetic waves and the induced voltages developed at the terminals of these elements. These voltages will be contaminated by the coupled energy between the elements of the array due to retransmission when the elements are close to each other. The results of this work illustrate that using this model in the cost function of the MUSIC algorithm results in DOA estimation of signals received by a receiving antenna array with very high resolution and very low RMSE when compared with another mutual coupling model.

Keywords- Antenna array, “Coupled Voltages to Uncoupled Currents” mutual coupling model, DOA estimation, MUSIC algorithm, and RMSE criteria.

I. INTRODUCTION

One main characteristic of an antenna array is mutual coupling. This phenomenon is inherent and unavoidable, especially when the array aperture size is small, i.e., when the elements are close to each other. Many of the array's fundamental characteristics, such as gain, directivity, and input impedance, are positively or negatively affected by the energy exchanged between the elements of the array when energized by an external source. Significant uses that are offered by antenna arrays, such as DOA estimation, rejecting interference signals, and others, are adversely affected by mutual coupling [1][2][3][4]. In an antenna array operating in receiving mode, each antenna receives a significant portion of the retransmitted energy from a nearby antenna in addition to the energy received from the impinging electromagnetic waves. As a result, the antenna's measurement will be contaminated and will not reflect the actual response to the received signal. Therefore, it is possible that several crucial antenna array-related variables were calculated incorrectly. For example, if the mutual coupling is not accurately accounted for or described in a receiving antenna array, the direction of arrival (DOA) of signals impinging on the array will be inaccurately assessed when the measurements are fed into DOA algorithms [5][6][7]. Therefore, it is essential to compensate for or counteract the effect of mutual coupling to obtain the true parameters.

For an N -element antenna array, the mutual coupling phenomenon is typically represented by an $N \times N$ matrix. The elements of this matrix are frequently complex values and they represent either the self-impedance and the mutual impedance between the elements of the array or the scattering parameters. There are several different models, which are expressing mutual coupling characteristic, are presented in the literature.

The concept of “Open Circuit Voltage” (OCV) model is presented in [9]. In this model, a receiving antenna array is represented by a bilateral network with $N + 1$ ports with the extra port being the one for the received signals. The basis of this concept has been developed based on circuit theory. Assuming that one antenna is loaded and the terminals of other antenna elements are open-circuited, a system of equations describing the energy exchange through the mutual coupling impedance between the elements of the array has been computed. Using the OCV model into the cost function of the MUSIC algorithm, a fairly DOA estimation is obtained from an affected steering matrix of a receiving antenna array due to mutual coupling [10]. However, the OCV model is criticized because the receiving antenna will still have an induced current even if its terminals are open-circuited. Therefore, it is assumed that this model does not adequately represent the corrected terms for mutual coupling [11][12]. On the other hand, a half-wave dipole antenna with a very thin diameter and open-circuited terminals carries negligible current when receiving a signal. The model “Receiving Mutual Impedance Method” (RMIM) is proposed in [12], and [13]. In contrast to the OCV model, the elements of the RMIM model, which involve receiving mutual impedances between the elements, are proposed under the condition of the closed-circuit terminals of the antennas. However, the receiving mutual impedance of this model is calculated experimentally using an array

of monopole antennas [13][14]. In [15], the effect of the retransmitted mutual impedance has been considered in addition to the commonly used mutual impedance. Therefore, N^2 simultaneous equations are required to calculate the unknown mutual coupling terms. Additionally, the authors of [15] do not demonstrate how to determine the mutual impedance formula for retransmission. The mutual coupling model in [16], known as the "full wave" model, has been widely recognized for accurately interpreting the mutual coupling phenomenon [17]. The reason for this recognition is that the model is based on the concept of the Method of Moment (MoM), which accurately accounts for the exchanged energy between the elements of the array and the element itself. Consequently, this paradigm remains unaffected by the state of an element's terminals. However, to use this model, it is necessary to have prior knowledge of the Direction of Arrival (DOA) of the received signals [16][18]. The model Multiple Antenna Induced EMF (MAIE), proposed in [18], formulates the mutual coupling matrix using the concept of the induced EMF method. It assumes a one-volt supply powering one antenna in the array with the other antennas terminated with the load impedance. However, using the term $1/I(0)$ as an antenna impedance results in an open-circuit voltage with a unit of volt-square rather than a unit of volt.

In [10], a new mutual coupling model was proposed. It is known as CVUC, which stands for Coupled Voltages to Uncoupled Currents. It was formulated to propose the relationship between the uncoupled currents induced in the elements of an antenna array operating in receiving mode and the load voltages developed at the terminals of these antennas. By considering the total components of the magnetic vector potential and the electric field components on the surface of a received antenna, the validity of Thevenin equivalent circuit to the receiving antenna array was proved. This work examines the effectiveness of this model when used in the cost function of a well-known direction of arrival algorithm, the MUSIC.

II. THE CVUC MODEL

The terminal voltages of loaded antennas in a receiving antenna array receiving signals are expressed in terms of the coupled currents induced in these antennas through the CVUC model as [10]:

$$\mathbf{v}_L = (\mathbf{A} + \mathbf{B}\mathbf{D}^{-1})\mathbf{I} = \mathbf{G}\mathbf{I} \tag{1}$$

where $\mathbf{v}_L = [v_{L,1} \ v_{L,2} \ \dots \ v_{L,N}]^T \in \mathbb{C}^{N \times 1}$ is the column vector of the terminal load voltages and $\mathbf{I} = [I_1 \ I_2 \ \dots \ I_N]^T \in \mathbb{C}^{N \times 1}$ is a column vector of the uncoupled currents. $\mathbf{G} = (\mathbf{A} + \mathbf{B}\mathbf{D}^{-1})$ is the CVUC model under consideration. The derivation of this model is based on electromagnetic theory and is represented by the Thevenin equivalent circuit. As a result, the couple-free currents, which are the response of the antenna elements to the received signals, can be extracted from the contaminated measurements of the array. Processing these uncoupled currents results in obtaining the true information of the received signals, such as the direction of arrival (DOA). The structures of the matrices \mathbf{A} , \mathbf{B} , and \mathbf{D} are as follows, respectively:

$$\mathbf{A} = \begin{bmatrix} Z_{11} + Z_{L,1} & 0 & 0 & \dots & 0 \\ 0 & Z_{22} + Z_{L,2} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & Z_{(N-1)(N-1)} + Z_{L,N-1} & & 0 \\ 0 & 0 & & & Z_{NN} + Z_{L,N} \end{bmatrix} \tag{2}$$

$$\mathbf{B} = \begin{bmatrix} -Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & -Z_{22} & \dots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \dots & -Z_{NN} \end{bmatrix} \tag{3}$$

$$\mathbf{D} = \begin{bmatrix} 1 & \frac{-Z_{12}}{Z_{11} + Z_{L,1}} & \dots & \frac{-Z_{1N}}{Z_{11} + Z_{L,1}} \\ \frac{-Z_{21}}{Z_{22} + Z_{L,2}} & 1 & \dots & \frac{-Z_{2N}}{Z_{22} + Z_{L,2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{-Z_{N1}}{Z_{NN} + Z_{L,N}} & \frac{-Z_{N2}}{Z_{NN} + Z_{L,N}} & \dots & 1 \end{bmatrix} \tag{4}$$

The complete proof for the system of equations in (1) can be found in [10]. Thus, the matrix \mathbf{G} maps the uncoupled currents to the coupled terminal voltages. Note that Z_{pp} and Z_{pq} in (2), (3), and (4) are respectively the self-impedance and the mutual impedance that were derived from the theory of the induced EMF [19]. For an array consisting two antennas, the matrix \mathbf{G} will be [10]:

$$\mathbf{G} = \begin{bmatrix} \frac{(z_{11}+z_{L,1})z_{L,1}z_{22}}{(z_{11}+z_{L,1})z_{22}-z_{12}z_{21}} & \frac{z_{22}z_{12}z_{L,1}}{(z_{11}+z_{L,1})z_{22}-z_{12}z_{21}} \\ \frac{z_{11}z_{21}z_{L,2}}{(z_{22}+z_{L,2})z_{11}-z_{12}z_{21}} & \frac{(z_{22}+z_{L,2})z_{L,2}z_{11}}{(z_{22}+z_{L,2})z_{11}-z_{12}z_{21}} \end{bmatrix} \quad (5)$$

The matrix \mathbf{G} in (5) has a Toeplitz structure. In the following sections and according to the pros and cons of the mutual coupling models mentioned in the previous section, the CVUC model will be evaluated by comparing it only against the OCV model since this model is considered as a good approximation for representing mutual coupling in receiving antenna array [20][21]. The uncoupled voltages (the open-circuit voltages) developed at the terminals of the elements in a receiving antenna array are related to the coupled voltages through the OCV model as [9]:

$$\mathbf{v}_{oc} = \mathbf{T}\mathbf{v}_L \quad (6)$$

where \mathbf{v}_{oc} , and \mathbf{v}_L are column vectors of the uncoupled and coupled voltages, respectively, of the array. The structure of the mutual coupling matrix $\mathbf{T} \in \mathbb{C}^{N \times 1}$ is:

$$\mathbf{T} = \begin{bmatrix} 1 + \frac{z_{11}}{z_L} & \frac{z_{12}}{z_L} & \dots & \frac{z_{1N}}{z_L} \\ \frac{z_{21}}{z_L} & 1 + \frac{z_{22}}{z_L} & \dots & \frac{z_{2N}}{z_L} \\ \dots & \dots & \dots & \dots \\ \frac{z_{N1}}{z_L} & \frac{z_{N2}}{z_L} & \dots & 1 + \frac{z_{NN}}{z_L} \end{bmatrix} \quad (7)$$

where z_{rs} is the transmitting mutual impedance between element r and element s of the array [4]. The assessment of CVUC model will consider the concept of DOA estimation and root mean square error (RMSE) criterion.

III. DOA ESTIMATION FROM COUPLED MEASUREMENTS

DOA estimation as a vital field of array processing has attracted numerous researchers since the beginning of wireless communications. By being capable of estimating the exact direction of incoming signals received by an antenna array, the most effective route for securely exchanging data during wireless communications can be easily determined. Also, increased data throughput and connected channels can be achieved. The effect of mutual coupling between array elements, especially when the aperture size of the array is small compared to the wavelength of the received signal, is a major problem among other problems associated with array processing. It is important that array measurements reflect only the response to received signals and lack the energy coupled between array elements due to retransmission. Therefore, the more accurate the interpretation or expressing the mutual coupling between the components of the array, the more precisely extracted the uncoupled values.

Let an antenna array consist of N elements and receive M narrowband signals. Also, let these elements be half wave dipole antennas distributed along the x -axis and vertically oriented and parallel to the z -axis with their centers located on the x -axis. This type of array is known as a Uniform Linear Array (ULA). Let the elements be equally spaced with an interelement spacing d (see figure 1). Assuming that the signals are received in the x - y plane, the measurement of this array will be [10]:

$$\mathbf{y}(t) = \mathbf{C}[\mathbf{F}(\phi)\mathbf{s}(t) + \mathbf{n}(t)] \quad (8)$$

where $\mathbf{y}(t) \in \mathbb{C}^{N \times 1}$ is the column vector of the measurement. $\mathbf{C} \in \mathbb{C}^{N \times N}$ is the mutual coupling matrix, $\mathbf{F}(\phi) = [\mathbf{f}(\phi_1) \mathbf{f}(\phi_2) \dots \mathbf{f}(\phi_M)] \in \mathbb{C}^{N \times M}$ is the steering matrix and it involves some information about the array's response to the received signals such as element gain and the DOA. In this work, $\mathbf{f}(\phi_m) \in \mathbb{C}^{N \times 1}$ is a vector that contains the response of each element in the array to the received signal m assuming the gain of each element is one. $\mathbf{s}(t) \in \mathbb{C}^{M \times 1}$ is a column vector of the strength of the received signals, and $\mathbf{n}(t) \in \mathbb{C}^{N \times 1}$ is the column vector of the noise. Equation (8) is valid when the noise is affected by the mutual coupling, such as cosmic noise. However, if the noise is not coupled, then (8) becomes [10][22][23][24]:

$$\mathbf{y}(t) = \mathbf{C}\mathbf{F}(\phi)\mathbf{s}(t) + \mathbf{n}(t) \quad (9)$$

The effect of the mutual coupling can be easily eliminated from the measurement in (8) by multiplying it by the inverse of \mathbf{C} , i.e., \mathbf{C}^{-1} if there is a prior knowledge of the mutual coupling (e.g., by array calibration). The covariance matrix is then calculated from the couple-free measurement, $\tilde{\mathbf{y}}(t) = \mathbf{C}^{-1}\mathbf{y}(t)$ as follows:

$$\begin{aligned} \mathbf{R} &= \tilde{\mathbf{y}}(t)\tilde{\mathbf{y}}(t)^H \\ &= [\mathbf{F}(\phi)\mathbf{s}(t) + \mathbf{n}(t)][\mathbf{F}(\phi)\mathbf{s}(t) + \mathbf{n}(t)]^H \end{aligned}$$

$$= \mathbf{F}(\phi)\mathbf{S}\mathbf{F}(\phi)^H + \mathbf{\Sigma} \quad (10)$$

where $(\cdot)^H$ denotes the Hermitian operation. $\mathbf{S} \in \mathbb{C}^{M \times M}$ and $\mathbf{\Sigma} \in \mathbb{C}^{N \times N}$ are, respectively, the signal covariance and the noise covariance matrices. In (10), it is assumed that the received signals and the noise are random variables with normal distribution and zero mean value [10][22][23][24]. In addition, the signals are uncorrelated with the noise. Note that the covariance matrix $\mathbf{R} \in \mathbb{C}^{N \times N}$ is positive definite matrix and all its eigen values are real and positive [25]. Eigen decomposing \mathbf{R} results into two orthogonal subspaces: the signal subspace $\mathbf{V}_s \in \mathbb{C}^{N \times M}$ and the noise subspace $\mathbf{V}_n \in \mathbb{C}^{N \times (N-M)}$, i.e.,

$$\mathbf{R} = \mathbf{V}_s \mathbf{\Lambda}_s \mathbf{V}_s^H + \mathbf{V}_n \mathbf{\Lambda}_n \mathbf{V}_n^H \quad (11)$$

In practice, \mathbf{R} can be obtained from:

$$\begin{aligned} \hat{\mathbf{R}} &= \frac{1}{q} \sum_{i=1}^q \mathbf{y}(t_i) \mathbf{y}(t_i)^H \\ &= \hat{\mathbf{V}}_s \hat{\mathbf{\Lambda}}_s \hat{\mathbf{V}}_s^H + \hat{\mathbf{V}}_n \hat{\mathbf{\Lambda}}_n \hat{\mathbf{V}}_n^H \end{aligned} \quad (12)$$

Where, $(\hat{\cdot})$ means estimated value. Equation (12) is valid provided that q , the number of measurement snapshots, is too large, i.e., $q \rightarrow \infty$ to obtain unbiased estimation.

In [26], the Multiple Signal Classification (MUSIC) algorithm has been proposed. This algorithm suggests a significant high-resolution technique for direction finding and spectral estimation. Estimating the DOAs of the received signals impinged on antenna array can be obtained by searching the peaks for the MUSIC cost function as:

$$p(\phi) = \frac{1}{\mathbf{f}(\phi)^H \hat{\mathbf{V}}_n \hat{\mathbf{V}}_n^H \mathbf{f}(\phi)} \quad (13)$$

The concept behind (13) is that the column space of the steering matrix $\mathbf{F}(\phi)$ is orthogonal to the noise subspace, i.e., $\mathbf{F}(\phi)^H \hat{\mathbf{V}}_n = \mathbf{0}$ with $\mathbf{0}$ is the zero matrix with size $M \times (N - M)$ [26].

If the measurement follows (9), i.e., there is no coupled noise between the elements, the method suggested for eliminating the mutual coupling in (8) will be not useful. Alternatively, (10) becomes:

$$\begin{aligned} \mathbf{R} &= \mathbf{y}(t) \mathbf{y}(t)^H \\ &= \mathbf{C}\mathbf{F}(\phi)\mathbf{S}\mathbf{F}(\phi)^H \mathbf{C}^H + \mathbf{\Sigma} \end{aligned} \quad (14)$$

and (13) becomes [10][22][23][24]:

$$p(\phi) = \frac{1}{\mathbf{f}(\phi)^H \mathbf{C}^H \hat{\mathbf{V}}_n \hat{\mathbf{V}}_n^H \mathbf{C} \mathbf{f}(\phi)} \quad (15)$$

Since under such a condition the orthogonality: $\mathbf{F}(\phi)^H \mathbf{C}^H \hat{\mathbf{V}}_n = \mathbf{0}$ satisfies.

IV. THE RMSE VALUE

The root mean square error (RMSE) is another criterion that may show whether the mutual coupling models in conjunction with DOA algorithms are effective. This criterion evaluates the proximity of the estimated value to the true value using the following relationship:

$$RMSE = \sqrt{\frac{\sum_{i=1}^L (\hat{\phi}_i - \phi_1)^2}{L}} \quad (16)$$

where $\hat{\phi}_i, \phi_1$ are the estimated and true DOA and L is the number of calculated runs. RMSE calculations in this work will be examined against a range of S/N for the models CVUC and OCV. The smaller the difference between the estimated and actual DOA, the better the performance of the mutual coupling model.

IV. SIMULATION RESULTS

In this section, several simulations will be run to determine the effectiveness of the CVUC model as a mutual coupling model in receiving antenna arrays. As previously stated, the concepts of DOA and RMSE will be used to evaluate the CVUC model compared to the OCV model. Let the array proposed in section III consists of six half wave dipoles with the first element located at the origin. The interelement spacing is $d = 0.5\lambda$, where λ is the wavelength of the received signals, see Figure 1. Two signals coming from the directions $\phi_1 = 60^\circ$ and $\phi_2 = 65^\circ$ with each signal having a signal to noise ratio $S/N = 30dB$ are impinged on the array. Figure 2

shows the MUSIC spectrum for these two signals. The number of snapshots used to obtain this plot was $K=1000$. The Figure depicts that using $\mathbf{C} = \mathbf{G}$, which is defined in (1), in (15) results in estimating the DOA of the received signal with high resolution much better than using $\mathbf{C} = \mathbf{T}^{-1}$. Figure 3 illustrates that reducing the aperture size of the array by reducing the interelement spacing between the elements of the

array to $d = 0.3\lambda$, has no effect on the resolution of the underlined DOAs when CVUC is considered. In contrast, the array fails to detect the DOAs when using OCV model for this case. Figure 4 shows that the CVUC model also outweighs the OCV model if it is used in the cost function of MUSIC algorithm to estimate the DOAs of signals coming from the end fire direction. However, it was required higher S/N ratio for the received signals to correctly detect the DOAs.

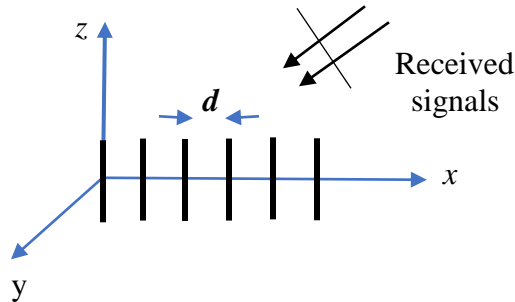


Figure 1: A ULA array consists of six half wave dipoles.

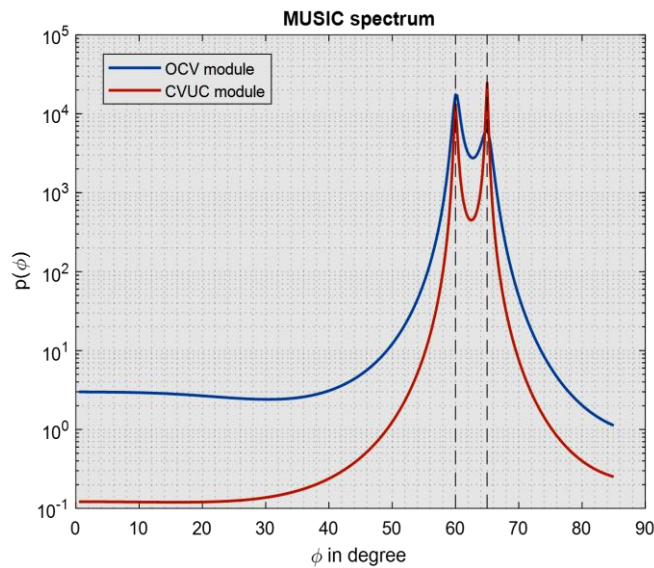


Figure 2: MUSIC spectrum for two signals coming from the directions $\phi_1 = 60^\circ$ and $\phi_2 = 65^\circ$ and impinging on the array shown in Figure 1 with $N=6$ and interelement spacing $d = 0.5\lambda$. Each signal is with $S/N = 30dB$ and 1000 snapshots are used for the measurements.

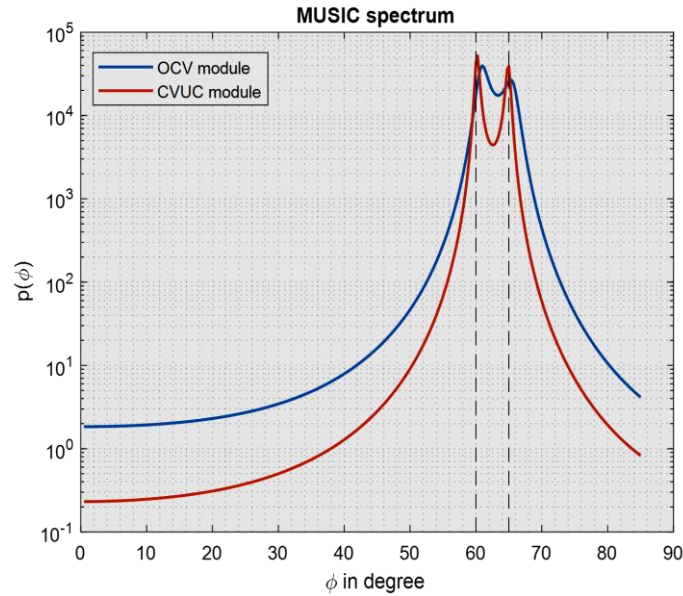


Figure 3: MUSIC spectrum for the same scenario used to plot Figure 2 but with $d = 0.3\lambda$

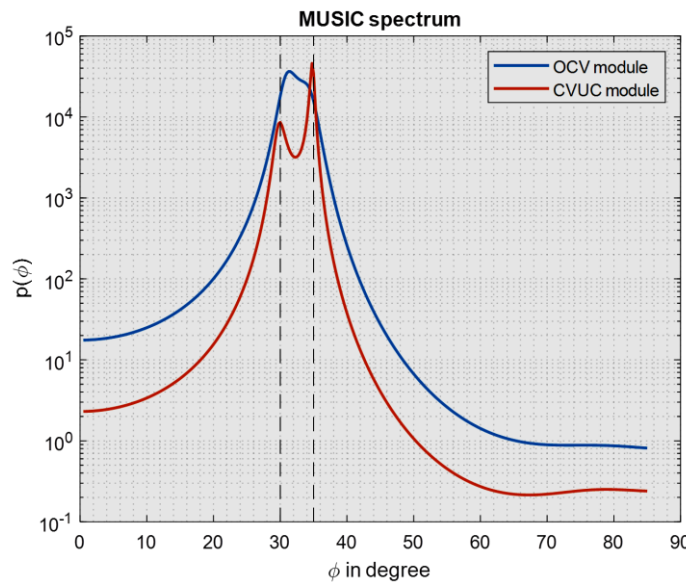


Figure 4: MUSIC spectrum for two signals coming from the directions $\phi_1 = 30^\circ$ and $\phi_2 = 35^\circ$ and impinging on the array shown in Figure 1 with $N=6$ and interelement spacing $d = 0.5\lambda$. Each signal is with $S/N = 35dB$ and 1000 snapshots are used for the measurements.

Figure 5 shows a plot for the RMSE versus S/N when a signal is coming from the direction $\phi_1 = 10^\circ$ and impinging on the same array of Figure 1. The interelement spacing $d = 0.5\lambda$. The number of runs were $L=100$ with the number of snapshots were $K=50$ per each run. Figure 6 also shows the RMSE plot versus S/N for the same scenario used for Figure 5, but the direction of the coming signal is $\phi_1 = 5^\circ$ and the interelement spacing is $d = 0.3\lambda$. Figures 5 and 6 show clearly that the difference between the true value and the estimated value of the DOA of the received signal will be significantly reduced when using CVUC model, regardless of the size of the array or the direction of received signal.

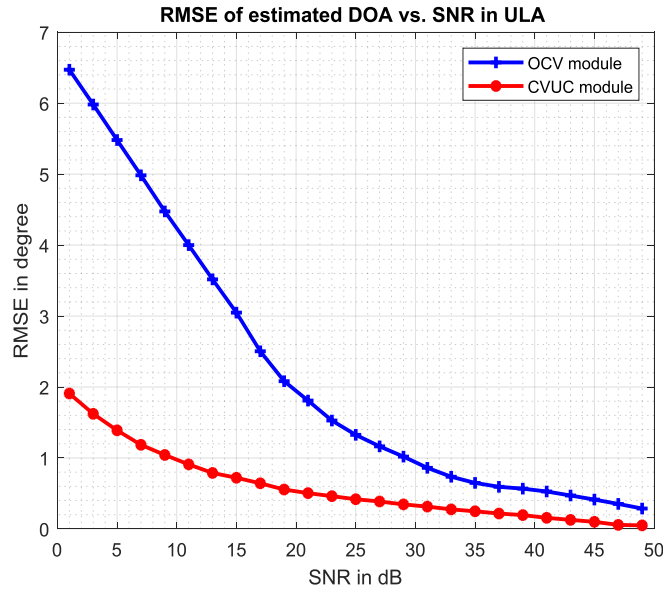


Figure 5: RMSE versus S/N for a signal coming from the direction $\phi_1 = 10^\circ$ and received by the array of Figure 1 with $d = 0.5\lambda$. $L=100$ and $K=50$ for each run

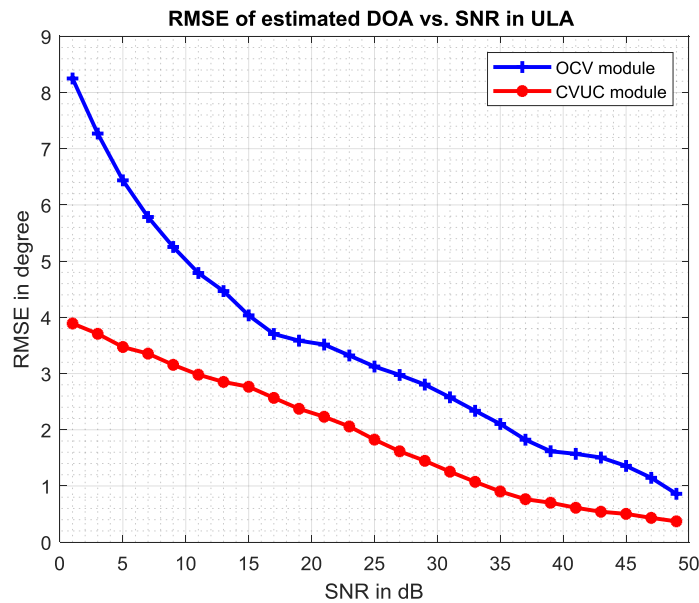


Figure 6: RMSE versus S/N for a signal coming from the direction $\phi_1 = 5^\circ$ and received by the array of Figure 1 with $d = 0.3\lambda$. $L=100$ and $K=50$ for each run

V. CONCLUSIONS

In this work, the effectiveness of the CVUC as a mutual coupling model in a receiving antenna array has been examined. The CVUC and OCV models are compared by investigating the performance of antenna array regarding DOA estimation and RMSE criteria. In conjunction with the MUSIC algorithm, it has been demonstrated that the model CVUC contributes to estimating the DOAs of signals received by the array with a much higher resolution than the OCV model under different conditions of the array structure and received signals. Additionally, compared to the OCV model, using the CVUC model yields a considerably smaller difference between the estimated and actual DOA.

Thus, CVUC can be regarded as an effective and sufficient interpretation of the mutual coupling phenomenon in receiving antenna array. Accordingly, some parameters of antenna array can be correctly calculated if a precise knowledge about the mutual coupling and its parameters are available.

Future works might be recommended for improvement of DOA estimation in the presence of mutual coupling effect. The first one is to formulate a DOA dependent CVUC model. This will provide an updated CVUC model where the elements of the receiving array are any type of antenna. Also, it can be worked to counteract the mutual coupling effect rather than using it in the cost function of the MUSIC algorithm. This might reduce the error associated with DOA calculations due to insufficient conditions required for an antenna array acting as direction finder.

REFERENCES

- [1] S. Roshani, S. Koziel, S.I. Yahya, M.A. Chaudhary, Y.Y. Ghadi, S. Roshani, L. Golunski, "Mutual Coupling Reduction in Antenna Arrays Using Artificial Intelligence Approach and Inverse Neural Network" Surrogates. *Sensors* 2023, 23, 7089. <https://doi.org/10.3390/s23167089>.
- [2] A. Ramos, T. Varum and J. N. Matos, "A Review on Mutual Coupling Reduction Techniques in mmWaves Structures and Massive MIMO Arrays," in *IEEE Access*, vol. 11, pp. 143143-143166, 2023, doi: 10.1109/ACCESS.2023.3343107.
- [3] T. Raj, R. Mishra, P. Kumar, A. Kapoor, "Advances in MIMO Antenna Design for 5G: A Comprehensive Review". *Sensors* 2023, 23, 6329. <https://doi.org/10.3390/s23146329>.
- [4] H. Singh, H. L. Sneha, and R. M. Jha, "Mutual coupling in phased arrays: A Review", *International Journal of Antennas and Propagation*, Hindawi Publishing Corporation, vol. 2013, Article ID 348123, 23 pages. <https://doi.org/10.1155/2013/348123>
- [5] M. Bensalem, and O. Barkat, "DOA estimation of linear dipole array with known mutual coupling based on ESPRIT and MUSIC". *Radio Science*, 57, e2021RS007294, 2022. <https://doi.org/10.1029/2021RS007294>
- [6] C. S. Ateşavcı, Y. Bahadırlar and S. Aldırmaz-Çolak, "DoA Estimation in the Presence of Mutual Coupling Using Root-MUSIC Algorithm," *2021 8th International Conference on Electrical and Electronics Engineering (ICEEE)*, Antalya, Turkey, 2021, pp. 292-298, doi: 10.1109/ICEEE52452.2021.9415938.
- [7] D. Li, Y. Jiang, X. Wu, and W.-P. Zhu. A Gridless Method for Doa Estimation Under the Coexistence of Mutual Coupling and Unknown Nonuniform Noise. *2020 IEEE 11th Sensor Array and Multichannel Signal Processing Workshop (SAM)*. 2020, pp. 1-5, doi: 10.1109/SAM48682.2020.9104255.
- [8] X. Zhang, T. Jiang, Y. Li, and Y. Zakharov. A Novel Block Sparse Reconstruction Method for DOA Estimation With Unknown Mutual Coupling. *IEEE Communications Letters*, vol. 23, no. 10, pp.1845-1848, 2019. doi:10.1109/lcomm.2019.2929384
- [9] I. J. Gupta and A. A. Ksienski, "Effect of Mutual Coupling on the Performance of Adaptive Arrays", in *IEEE Transactions on Antennas and Propagation*, vol. AP-31, no.5, September 1983. doi: 10.1109/TAP.1983.1143128.
- [10] Jasem, R. A. (2020). High Resolution Direction of Arrival Estimation with Switched Active Switched Parasitic Antenna Arrays, Doctoral dissertation, Curtin University.
- [11] N. Parhizgar, A. Alighanbari, M.-A. Masnadi-Shirazi, and A. Sheikhi., "A Modified Decoupling Scheme for Receiving Antenna Arrays with Application to DOA Estimation", *International Journal of RF and Computer-Aided Engineering*, vol. 23, no. 2, pp. 246-259, 2012. <https://doi.org/10.1002/mmce.20671>.
- [12] H. Lui, H. T. Hui, and M. S. Leong "A note on the Mutual-Coupling Problems in Transmitting and Receiving Antenna Arrays", in *IEEE Antennas and Propagation Magazine*, vol. 51, no. 5, pp. 171-176, 2009. doi: 10.1109/MAP.2009.5432083.
- [13] Y. Yu and H. T. Hui, "Design of Mutual Compensation Network for a Small Receiving Monopole Array", in *IEEE Transactions on Microwave Theory and Techniques*, vol. 59, no. 9, pp. 2241-2245, 2011. doi: 10.1109/TMTT.2011.2160728.
- [14] H. T. Hui and S. Lu, "Receiving Mutual Impedance between Two Parallel Dipole Antennas", *IEEE Region 10 Conference*, pp. 1-4, 2006. doi: 10.1109/TENCON.2006.343692.
- [15] H. Yamada, Y. Ogawa, and Y. Yamaguchi, "Mutual Coupling Compensation in Array Antenna for High-Resolution DOA Estimation", *Proceedings of ISAP2005*, Korea, 2005.
- [16] R. S. Adve and T. K. Sarkar, "Compensation for the effects of mutual coupling on direct data domain adaptive algorithms," in *IEEE Transactions on Antennas and Propagation*, vol. 48, no. 1, pp. 86-94, Jan. 2000, doi: 10.1109/8.827389.
- [17] S. Henault and Y. M. M. Antar, "Comparison of Various Mutual Coupling Compensation Methods in Receiving Antenna Arrays", in *IEEE Antennas and Propagation Society International Symposium*, pp. 1-4, June 2009. doi: 10.1109/APS.2009.5172379.
- [18] Henault, S., Antar, Y.M., Rajan, S., Inkol, R.J., & Wang, S., "The Multiple Antenna Induced EMF Method for The Precise Calculation of The Coupling Matrix in A Receiving Antenna Array". *Progress in Electromagnetics Research M*, 8, 103-118, 2009. doi: 10.2528/PIERM09062309.
- [19] C. A. Balanis, *Antenna theory: Analysis and design*, 4th ed., John Wiley & Sons, 2016.
- [20] D.-W. Kim, and S. Nam. "Mutual Coupling Compensation in Receive-mode Antenna Array Based on Characteristic Mode Analysis", in *IEEE Transactions on Antennas and Propagation*, vol. 66, no. 12, pp. 7434-7438, December 2018. doi:10.1109/tap.2018.2869431.

- [21] A. Tariq, S. Khattak, H. Munsif, S. Razzaq, and Irfanullah. "Linear Pattern Correction Technique for Compensating the Effects of Mutual Coupling and Deformation in Wedge-Shaped Conformal Antenna Arrays", *ACES Journal*, vol. 36, no. 05, pp. 533–541, Jul. 2021. doi: [10.47037/2020.ACES.J.360507](https://doi.org/10.47037/2020.ACES.J.360507).
- [22] Y. Fang, S. Zhu, H. Wang, and C. Zeng, "DOA estimation via ULA with mutual coupling in the presence of non-uniform noise", *Digital Signal Processing*, vol. 97, 2020, 102612, doi:[10.1016/j.dsp.2019.102612](https://doi.org/10.1016/j.dsp.2019.102612).
- [23] L. Li, X. Wang, X. Lan, G. Xu, L. Wan, "Reweighted Off-Grid Sparse Spectrum Fitting for DOA Estimation in Sensor Array with Unknown Mutual Coupling". *Sensors* 2023, 23, 6196. <https://doi.org/10.3390/s23136196>.
- [24] P. Chen, Z. Cao, Z. Chen, L. Liu, M. Feng, "Compressed Sensing-Based DOA Estimation with Unknown Mutual Coupling Effect". *Electronics* 2018, 7, 424. <https://doi.org/10.3390/electronics7120424>.
- [25] R. A. Horn and C. R. Johnson, *Matrix Analysis*, 2nd Edition, 2013, Cambridge University Press, New York, USA.
- [26] R. O. Schmidt, "Multiple Emitter Location and Signal Parameter estimation," in *IEEE Trans. Antennas and Propagation*, vol. 34, no. 3, pp. 276-280, March 1986. doi: [10.1109/TAP.1986.1143830](https://doi.org/10.1109/TAP.1986.1143830).

AUTHOR

Rabah Abduljabbar Jasem, PhD in Electrical Engineering (2020) from Curtin University, Perth, Australia. MSc. (2005) and BSc. (1984) in Electrical Engineering from the Department of Electrical Engineering, Engineering college, University of Baghdad. Email: rabah_alobaidi5@yahoo.com